Analysis of Lower Limb Key Point Displacement in the Frontal Plane

Sergey F. Yatsun, Alina A. Sidorova

Abstract—**This paper presents the results of an experimental study of the trajectories of lower limb key points in the frontal plane at different walking speeds on a treadmill. The aim of the study was to identify patterns of lateral oscillations and deviations necessary for maintaining walking stability, which is critically important for the design of biomechanically correct rehabilitation systems. The experiment was conducted with three subjects of different anthropometric profiles at three speed modes (0.14 m/s, 0.28 m/s, and 0.42 m/s). The methodology included motion video capture, trajectory digitization using OpenSourcePhysicsTracker software, and subsequent mathematical data processing. To transition from discrete coordinates to continuous analytical models, the approximation method using Fourier trigonometric series was applied. The results, presented as a series of graphs, demonstrate characteristic changes in the trajectories of the hip, knee, and ankle joints depending on speed: reduction of vertical deviation, enhancement of figure-eight pattern motion, rounding of trajectories, and symmetric multidirectionality of paired joints. The hierarchical principle of coordination was confirmed: in-phase movement of the proximal pelvic joints is combined with antiphase activity of paired distal joints, both between limbs and within a single limb. The obtained data and their analytical description can serve as a foundation for creating more accurate biomechanical models and improving control algorithms for rehabilitation exoskeletons and mechanotherapy systems.**

Keywords—**gait biomechanics, Fourier series approximation, frontal plane, rehabilitation robotics.**

I. INTRODUCTION

In Russia and other CIS countries, there is an increase in the number of people suffering from neurological diseases and traumatic injuries that impair the functions of the human musculoskeletal system [1], [2]. Such changes in the structure of disease prevalence place an additional burden on the healthcare sector, exacerbating the issue of accessibility and quality of long-term rehabilitation [3], [4].

To reduce the burden on medical professionals, modern medicine is actively developing the field of rehabilitation exoskeletons and mechanotherapy [5]–[7]. These technologies are designed to compensate for the shortage of rehabilitation resources by providing reproducible and objectively controlled training [8], [9].

Based on the above, engineers face the task of ensuring the biomechanical correspondence of the movement of rehabilitation systems [10]–[12].

In early works on biomechanics, gait was studied primarily in the sagittal plane [13]–[15]. However, a full understanding of gait mechanisms requires consideration of movements in the frontal plane as well — lateral oscillations and deviations that ensure stability [16], [17].

The aim of this paper is to study human gait patterns in the frontal plane under different speed conditions.

II. EXPERIMENTAL METHODOLOGY

To ensure reproducibility of results and minimize errors, a unified data processing methodology was developed, as shown in Fig. 1.

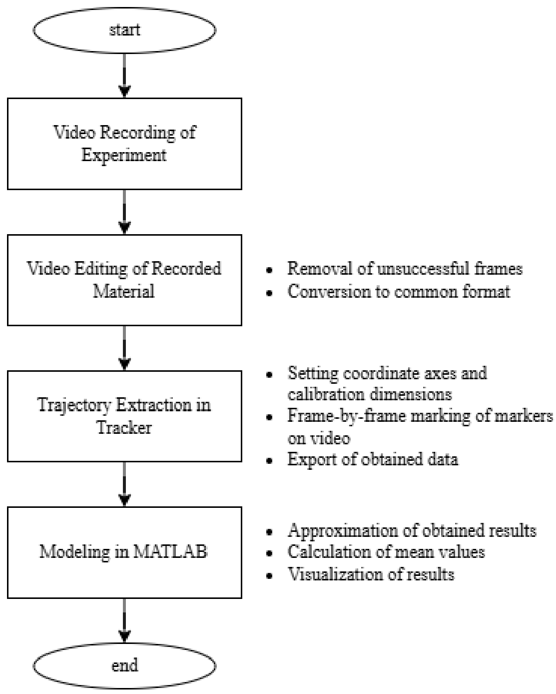

Fig. 1. Block diagram of the data processing algorithm

The process includes four key stages: video recording, editing of the obtained material, coordinate tracking in the Tracker software, and mathematical modeling in the MATLAB environment.

For video recording, a camera with a frame rate of 60 Hz and a resolution of 1920×1080 pixels (FHD) was used. The experimental procedure is described below in the corresponding section. Considering the specificity of the studied movements and in order to reduce the labor intensity of subsequent processing, the original video frame rate was reduced to 30 frames per second at the video editing stage. For this purpose, the Shotcut video editor was used [18].

Manuscript received March 11, 2026. This work was supported by the state task No. 075-03-2026-489.

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The selected sampling rate is sufficient for correctly describing the kinematics at the given speed modes and does not lead to the loss of significant motion information. At the final stage of video editing, all files were converted to a unified format, ensuring correct loading and processing in the Tracker environment.

The tracking process in the Tracker program consists of frame-by-frame selection of markers corresponding to key anatomical points throughout the entire movement cycle. A detailed description of the digitization procedure is provided in Section IV. As a result of processing, raw data arrays are formed, containing discrete coordinate values of the tracked points at each moment in time. To ensure compatibility with subsequent stages of analysis, the obtained arrays were exported to the Excel spreadsheet processor, after which they were imported into the MATLAB environment for further mathematical processing. The mathematical modeling process is presented in Section V.

III. EXPERIMENTAL PROCEDURE

To conduct the study, video recording of the participants walking on a moving surface under different speed conditions was performed. The experimental setup is shown in Fig. 2.

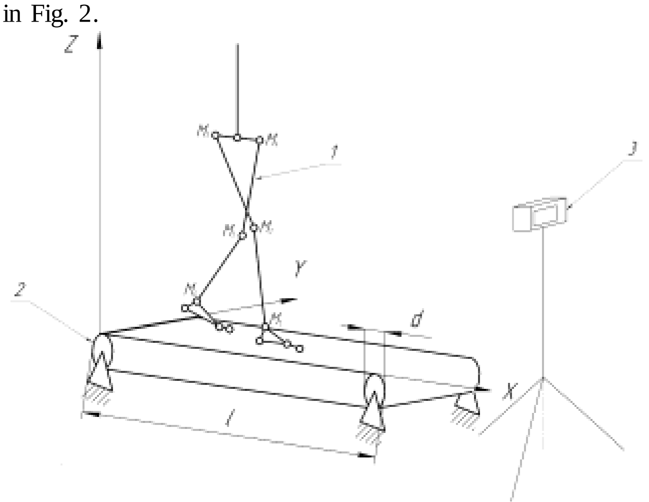

Fig. 2. Diagram of the experimental setup for gait analysis: 1 — subject, 2 — treadmill, 3 — video camera, M_3, M_4 — hip joints, M_2, M_5 — knee joints, M_1, M_6 — ankle joints

The main characteristics of the treadmill were as follows:

- Belt inclination angle: 0–3°
- Belt length: 1.03 m
- Roller diameter: 0.36 m
- Speed range: 0–1.5 km/h

Each participant performed test walks on the treadmill for 30 seconds at each of the three speed modes. Before recording began, participants were given 15 seconds to adapt to the set speed. Masking tape and paint were used to mark key points. Video recording was performed using a camera with a frame rate of 60 Hz, mounted on a tripod strictly perpendicular to the frontal plane of motion at a distance of 40–60 cm, depending on the subject's height. Lighting was constant and diffused to prevent glare on the markers.

In our experiment, three walking speeds were defined:

1. Slow: $0 \le v \le 0.14(m/s)$;
2. Medium: $0.14(m/s) \le v \le 0.28(m/s)$;
3. Fast: $0.28(m/s) \le v \le 0.42(m/s)$;

Anthropometric data of the participants are presented in Table 1.

Table 1. Participant data

№	Gender	Height, cm	Weight, kg	Thigh length, cm	Shin length, cm	Ankle height, cm
1	m	180	65	41	40	10
2	m	186	80	43	45	9
3	f	163	45	40	39	5
4	f	168	70	43	42	7
5	f	168	64	48	45	7
6	m	186	83	49	48	8
7	m	193	76	47	45	8
8	m	188	105	45	45	7
9	m	179	62	43	46	8
10	m	178	58	42	38	7

To ensure the required treadmill speed, the following calculations were performed.

Considering the length of the treadmill belt, the total loop length was determined:

$$L=2 \cdot l; \quad (1)$$

$L = 2 \cdot 1.03 = 2.06m.$

For the desired treadmill speed of 0.14 m/s, the time for one full revolution was calculated as:

$$T = \frac{L}{V}; \quad (2)$$

$$T = \frac{2.06}{0.14} = 14.82 \approx 15s.$$

Consequently, for speeds of 0.28 m/s and 0.42 m/s, the corresponding periods were 10 s and 5 s, respectively.

IV. VIDEO DATA DIGITIZATION

Prior to the digitization process, the video clips were processed using the method described in Section II. This preliminary processing included frame rate normalization to 30 fps, removal of irrelevant segments, and format conversion to ensure compatibility with the Tracker software. To capture the trajectories of key points from the processed video material, the Tracker software was used [19]. After importing the video clips, a calibration procedure was performed: the coordinate origin was established, the calibration scale was set using a reference object of known dimensions, and the key anatomical points for trajectory tracking were identified. The process is shown in Fig. 2. When recording walking in the frontal plane, the treadmill belt width, equal to 0.37 m, was used as the calibration scale, providing a reliable reference for converting pixel coordinates to real-world measurements. As a result of digitization, data arrays were obtained representing the time dependence of the coordinates of lower limb key point displacement. These raw data arrays contained discrete coordinate values for each tracked point at each time instant corresponding to the video frame rate.

Fig. 3 Digitization process of frontal plane walking in the Tracker program

V. MATHEMATICAL MODELING

The obtained discrete coordinates of key points require transformation into a continuous analytical description for biomechanical analysis and application in control systems. To address this task, the experimental trajectories were approximated using Fourier series expansion [20], [21]. The choice of this method is justified by the periodic nature of the gait cycle, which corresponds to the mathematical structure of a trigonometric series. For coefficient fitting, the MATLAB software environment with the Curve Fitting Toolbox was used [22]. The general formula of the Fourier trigonometric series is presented below:

$$f(t) = a_0 + \sum_{q=1}^{N} (a_q \cos(q\omega t) + b_q \sin(q\omega t)) \qquad (3)$$

where a_0 – is the constant term (mean value of the function over the period); a_q, b_q – are the linear coefficients of the Fourier series, calculated for each harmonic; N – is the series order; q – is the harmonic number; ω – is the fundamental angular frequency.

To select the number of harmonics (series order), both numerical and empirical methods were used. When working with the mathematical tool, during coefficient calculation, a preview of the future function is automatically generated. Too few harmonics do not always adequately describe the trajectory; however, an excessive number causes the model to begin fitting the noise in the data. For numerical evaluation, the program automatically calculates the sum of squared errors, coefficient of determination, adjusted R-squared, and root mean square error.

To identify general patterns in the displacement trajectories, a transition from individual data to averaged values was performed. For this purpose, the obtained values were reduced to the arithmetic mean. This method allows leveling individual anthropometric differences between subjects and random fluctuations in trajectories, highlighting common movement patterns; however, averaging may conceal individual movement characteristics that could be important for personalized rehabilitation. To ensure data comparability across different subjects, the time series were synchronized to the beginning of the gait cycle (see Fig. 5). In this study, the gait cycle is defined as the time interval from the moment the subject's heel lifts off the surface until it lifts off again. Despite significant differences in quantitative values, the trajectories qualitatively share a common structure. The average coordinates were calculated using the following formula:

$$\bar{y}_{cp} = \frac{\sum_{i=1}^{n} y_i}{n}; \qquad \bar{z}_{cp} = \frac{\sum_{i=1}^{n} z_i}{n}. \qquad (4)$$

This n – is the total number of experimental participants;

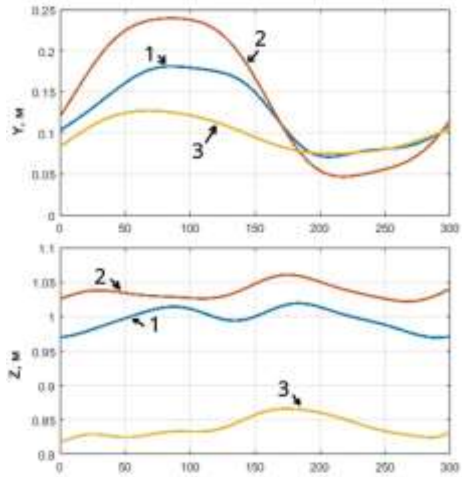

Fig. 4. Gait phase synchronization using hip joint displacement of experimental participants: 1 — first participant, 2 — second participant, 3 — third participant

Figs. 5–7 present the displacement trajectories of the hip, knee, and ankle joints, respectively, at a speed of 0.14 m/s. Arrows indicate the direction of joint movement.

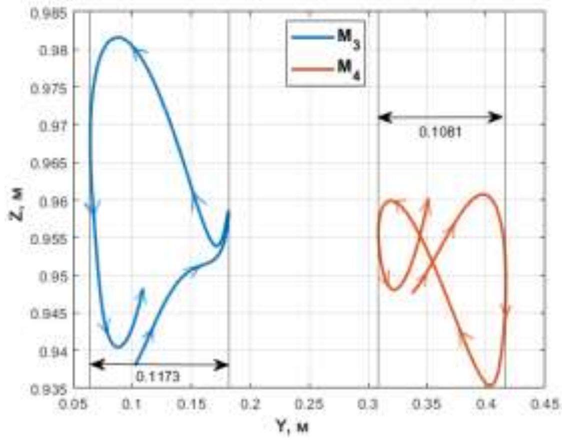

Fig. 5. Displacement of hip joints in the frontal plane at a speed of 0.14 m/s: M_3 – right joint, M_4 – left joint

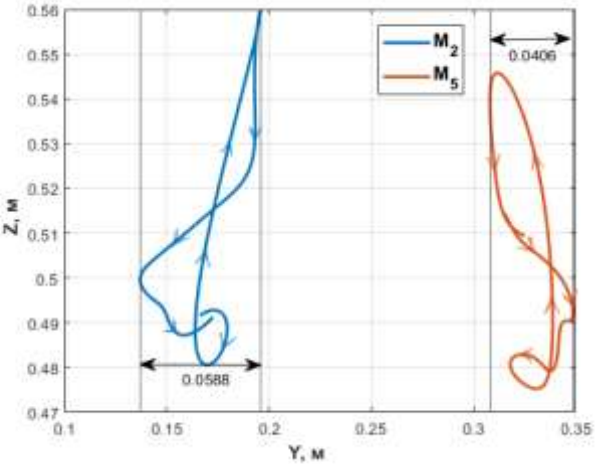

Fig. 6 Displacement of knee joints in the frontal plane at a speed of 0.14 m/s: M_2 – right joint, M_5 – left joint

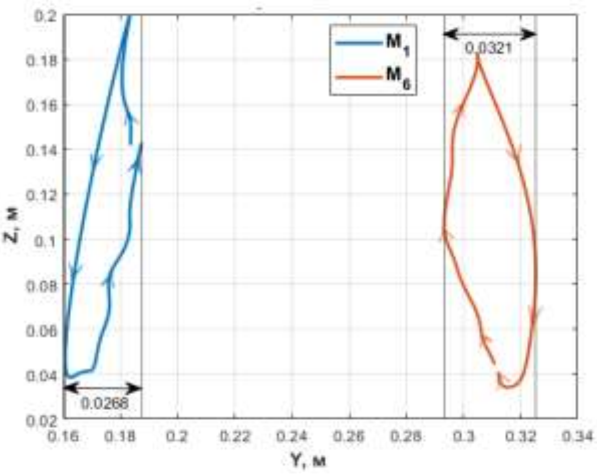

Fig. 7 Displacement of ankle joints in the frontal plane at a speed of 0.14 m/s: M_1 – right joint, M_6 – left joint

Figs. 8–10 present the displacement trajectories of the hip, knee, and ankle joints, respectively, at a speed of 0.28 m/s. Arrows indicate the direction of joint movement.

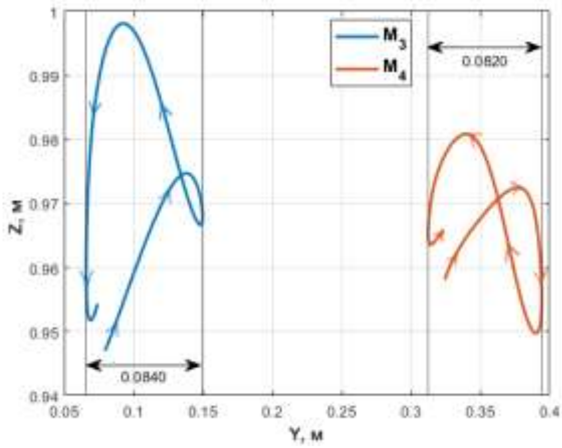

Fig. 8 Displacement of hip joints in the frontal plane at a speed of 0.28 m/s: M_3 – right joint, M_4 – left joint

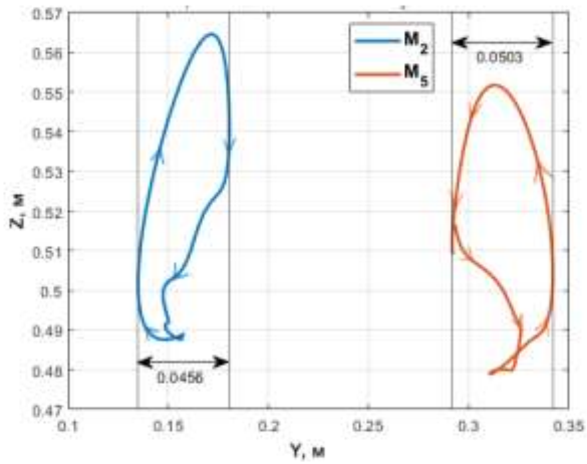

Fig. 9 Displacement of knee joints in the frontal plane at a speed of 0.28 m/s: M_2 – right joint, M_5 – left joint

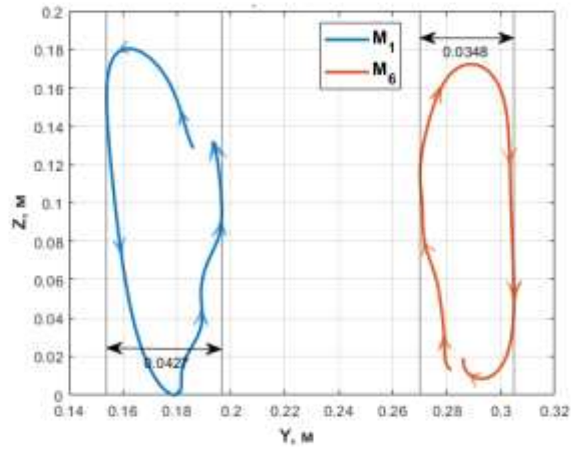

Fig. 10 Displacement of ankle joints in the frontal plane at a speed of 0.28 m/s: M_1 – right joint, M_6 – left joint

Figs. 11–13 present the displacement trajectories of the hip, knee, and ankle joints, respectively, at a speed of 0.42 m/s. Arrows indicate the direction of joint movement.

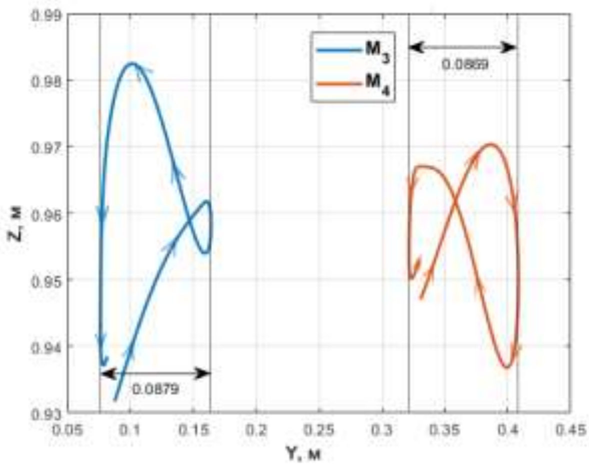

Fig. 11 Displacement of hip joints in the frontal plane at a speed of 0.42 m/s: M_3 – right joint, M_4 – left joint

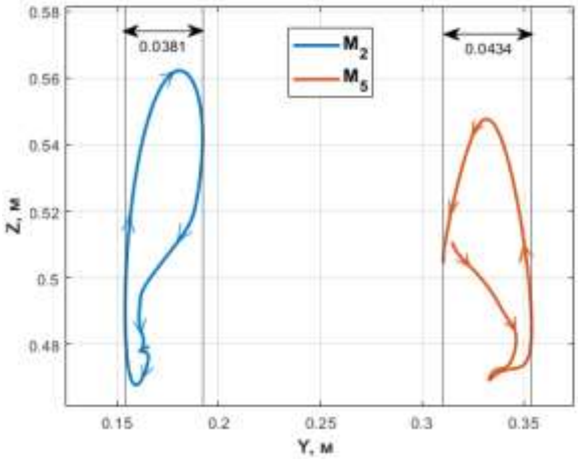

Fig. 12 Displacement of knee joints in the frontal plane at a speed of 0.42 m/s: M_2 – right joint, M_5 – left joint

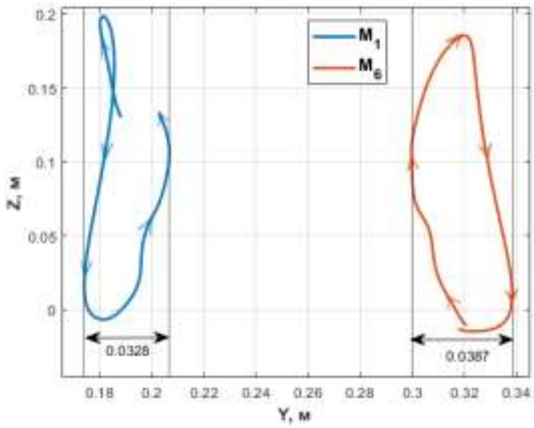

Fig. 13 Displacement of ankle joints in the frontal plane at a speed of 0.42 m/s: M_1 – right joint, M_6 – left joint

VI. ANALYSIS OF THE OBTAINED RESULTS

Based on the presented graphs, a table was compiled summarizing the data on the maximum displacement of key joints along the Y-axis. The data are presented in Table 2.

Table 2 Maximum joint displacements along the Y-axis

V, m/s	M_1, m	M_2, m	M_3, m	M_4, m	M_5, m	M_6, m
0.14	0.027	0.059	0.117	0.108	0.041	0.032
0.28	0.043	0.046	0.084	0.082	0.050	0.035
0.42	0.033	0.038	0.088	0.087	0.043	0.039

The observed decrease in the horizontal displacement of the hip joints when transitioning from slow to medium speed is consistent with the well-known biomechanical principle of increasing walking efficiency by minimizing vertical oscillations of the center of mass [23]–[25]. The increase in amplitude in the distal joints (knee and ankle) at medium speed may be explained by the need to increase support stiffness and enhance push-off to compensate for increased inertial forces. The subsequent slight reverse change in amplitudes at high speed requires verification on a more representative sample but may indicate a transition to a different gait pattern, close to fast walking with an emphasis on movements in the sagittal plane.

The complication of the hip joint trajectories, manifested in the formation of a pronounced "figure-eight" pattern, is a direct consequence of increased torsional moment in the pelvic region during gait acceleration. This pattern represents a compensatory mechanism aimed at dissipating excess angular energy and stabilizing the torso. This phenomenon confirms the principle of energy optimization of locomotion, in which the body seeks to minimize metabolic costs by redistributing mechanical work between proximal and distal segments [26]–[28]. The simultaneous smoothing and rounding of the knee and ankle joint trajectories indicates a decrease in the third derivative of acceleration (jerk), which is characteristic of a smoother and, consequently, more energy-efficient mode of neuromuscular control provided by stabilizer muscles.

Gait kinematics demonstrates hierarchical coordination. The hip joints move synchronously. At the same time, the knee and ankle joints implement two levels of antiphase activity: both contralateral limbs and joints within the same limb work in opposite directions.

The observed trajectory non-closure (hysteresis/drift) suggests compensatory oscillations of the body's center of mass, generated by postural control mechanisms to maintain dynamic stability. To quantify this in the frontal plane, deviations between gait cycle start and end points were calculated separately for the vertical (Z) and horizontal (Y) axes, followed by determination of total linear deviation.

The deviation relative to the axes was calculated as follows:

$$\delta Y = Y_{start} - Y_{end};$$
$$\delta Z = Z_{start} - Z_{end}. \tag{4}$$

The total absolute trajectory deviation in the plane was calculated using the Pythagorean theorem.:

$$\|\delta\| = \sqrt{\delta Y^2 + \delta Z^2}. \tag{5}$$

The calculation of deviations for the right leg is presented in Table 3.

Table 3 Deviations for the right leg

Joints	V, m/s	δY, m	δZ, m	||δ||, m
M1	0,14	-0,0039	-0,0012	0,00408
	0,28	-0,0073	-0,0036	0,008139
	0,42	-0,0147	-0,0033	0,015066
M2	0,14	-0,0063	3,06E-04	0,006307
	0,28	-0,0014	-6,89E-04	0,00156
	0,42	-0,0054	-0,0011	0,005511
M3	0,14	-0,007	-0,0103	0,012454
	0,28	0,005	-0,0074	0,008931
	0,42	0,0057	-0,0068	0,008873

VII. CONCLUSION

As a result of the conducted experimental study, the trajectories of the key lower limb joints in the frontal plane for three walking speed modes were obtained and mathematically described (using Fourier series approximation). The following main regularities were established:

The amplitude of lateral oscillations of the hip joints decreases with increasing speed from slow to medium, while the amplitude of the knee and ankle joints increases in this mode.

The shape of the trajectories qualitatively changes: in the hip joints, the component forming a "figure-eight" pattern intensifies, while the trajectories of the distal joints become more rounded and smoother.

The hierarchical principle of coordination was confirmed: in-phase movement of the proximal pelvic joints is combined with antiphase activity of paired distal joints, both between limbs and within a single limb.

The main limitation is the small sample size (n=3), which does not allow for statistically significant generalizations.

The obtained analytical trajectory models and identified patterns are of practical value for developers of rehabilitation robotics, as they can serve as reference trajectories for position control systems or as basic laws for algorithms based on synergetic control principles.

REFERENCES

[1] Federal State Statistics Service (Rosstat). (2023). *Official statistics*. [Online]. Available: https://rosstat.gov.ru/folder/13721. Accessed: Feb. 9, 2026.

[2] K. V. Antonova and T. S. Shishkina, "The problem of cerebro-osseous interactions in the clinical setting of neurological diseases," *Effective Pharmacotherapy*, vol. 21, no. 17, pp. 36–44, 2025. doi: 10.33978/2307-3586-2025-21-17-36-44.

[3] U. A. Malashenko, "Social adaptation of people with disabilities in modern society," *Scientific Proceedings of Moscow University for the Humanities*, no. 4, 2024.

[4] N. M. Dranichnikova, E. V. Kuftyak, and O. M. Solomatova, "Organizational model of territorial accessibility of comprehensive rehabilitation for disabled children with physical impairments," *Bulletin of Kostroma State University. Series: Pedagogy. Psychology. Sociokinetics*, no. 1, 2009.

[5] D. I. Safarov, E. D. Tistsov, and S. F. Yatsun, "Modeling of human-rehabilitation exoskeleton interaction," *International Journal of Open Information Technologies*, vol. 12, no. 4, 2024.

[6] E. I. Borzenko, D. S. Zhdanov, and R. E. Makarov, "Rehabilitation wrist exoskeleton for restoring lost functions of the upper limb," *Robotics and Technical Cybernetics*, no. 3, 2025.

[7] A. A. Vorobyov, O. A. Zasypkina, P. S. Krivonozhkina, A. V. Petrukhin, and A. M. Pozdnyakov, "Exoskeleton — state of the problem and prospects for implementation in the system of habilitation and rehabilitation of disabled people (analytical review)," *Bulletin of Volgograd State Medical University*, no. 2(54), 2015.

[8] D. A. Kaneva, T. Yu. Tararaeva, A. V. Breusov, and L. V. Maksimenko, "The problem of medical staff shortage in Russian healthcare: causes and solutions (literature review)," *Modern Problems of Healthcare and Medical Statistics*, no. 1, 2024.

[9] A. A. Zolotov and S. V. Panikarova, "Staff shortage in regional healthcare: myths, causes, solutions," *Vocational Education and Labor Market*, no. 1(60), 2025.

[10] S. G. Polyanskaya and T. V. Telyatnikova, "Digitalization of the Russian healthcare system as a promising direction for the development of the digital economy," *Bulletin of Siberian Institute of Business and Information Technologies*, no. 3, 2024.

[11] D. Dib and I. V. Merkuryev, "Optimal control method for a lower limb exoskeleton with elastic elements," *Advanced Engineering Research (Rostov-on-Don)*, no. 3, 2025.

[12] N. G. Kurakova and F. A. Kurakov, "Problems of bringing domestic high-tech products to the global market using the example of medical robotics," *Russia: Trends and Development Prospects*, no. 12-2, 2017.

[13] S. F. Yatsun, G. A. Fursov, and A. A. Sidorova, "Mathematical modeling of foot movement when walking on a moving horizontal surface," *Proceedings of Southwest State University. Series: Control, Computer Engineering, Informatics. Medical Instrument Engineering*, vol. 15, no. 4, pp. 89–106, 2025. doi: 10.21869/2223-1536-2025-15-4-89-106.

[14] M. I. Pikalov, D. A. Petrov, and E. N. Revnyakov, "Study of human lower limb kinematics during locomotion," *Bulletin of Science*, vol. 1, no. 7(7), 2018.

[15] L. I. Myakotina, S. V. Gyulnazarova, and N. V. Smirnova, "Biomechanical assessment of the musculoskeletal system functioning in chronic injuries of the knee joint extensor apparatus," *Genius of Orthopedics*, no. 4, 2008.

[16] M. W. Whittle, "Clinical gait analysis: A review," *Human Movement Science*, vol. 15, no. 3, pp. 369–387, Jun. 1996.

[17] A. L. Hof, "The 'extrapolated center of mass' concept suggests a simple control of balance in walking," *Human Movement Science*, vol. 27, no. 1, pp. 112–125, Feb. 2008.

[18] Shotcut, Version 23.11.21 [Computer software]. Available: https://www.shotcut.org. Accessed: Feb. 9, 2026.

[19] OpenSourcePhysicsTracker [Computer software]. Available: https://opensourcephysics.github.io/tracker-website/. Accessed: Feb. 9, 2026.

[20] Z. Zhao, *Mathematical Methods for Physicists*, 7th ed.

[21] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, 3rd ed. Cambridge, U.K.: Cambridge Univ. Press, 2007, ch. 12–13.

[22] Curve Fitting Toolbox [Computer software]. Natick, MA, USA: MathWorks. Available: https://www.mathworks.com/products/curvefitting.html. Accessed: Feb. 9, 2026.

[23] R. M. Alexander, "Energy-saving mechanisms in walking and running," *Journal of Experimental Biology*, vol. 160, no. 1, pp. 55–69, 1991.

[24] A. D. Kuo, "The six determinants of gait and the inverted pendulum analogy: A dynamic walking perspective," *Human Movement Science*, vol. 26, no. 4, pp. 617–656, Aug. 2007.

[25] M. S. Orendurff et al., "The effect of walking speed on center of mass displacement," *Journal of Rehabilitation Research and Development*, vol. 41, no. 6A, pp. 829–834, 2004.

[26] V. M. Zatsiorsky and M. L. Latash, "On the concept of 'dynamic synergies' and their relation to the problem of motor redundancy," *Experimental Brain Research*, vol. 183, no. 1, pp. 155–169, Nov. 2007.

[27] J. M. Donelan, R. Kram, and A. D. Kuo, "Mechanical and metabolic determinants of the preferred step width in human walking," *Proceedings of the Royal Society B: Biological Sciences*, vol. 268, no. 1480, pp. 1985–1992, Oct. 2001.

[28] D. G. Thelen and F. C. Anderson, "Using computed muscle control to generate forward dynamic simulations of human walking from experimental data," *Journal of Biomechanics*, vol. 39, no. 6, pp. 1107–1115, 2006.

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