Compensation of input and output disturbances for MIMO discrete-time systems with unmeasured state vector

Cong Vinh Tu, Natalia Dudarenko

***Abstract*— This paper addresses the challenge of compensating for external disturbances in a class of discrete-time linear multi-channel systems subject to both input and output disturbances. It is assumed that while the external signals have known parameters, the system states and external disturbance states cannot be directly measured. To address this challenge, the approach combines Francis' equation with a discrete-time PID observer. A full-order observer, referred to as the Discrete-Time PID Observer, is developed to estimate the state vectors. Based on Francis's equation, a regulator is designed to effectively control the output signal to track the reference signal. The effectiveness of the proposed solution is validated through MATLAB Simulink simulations.**

***Keywords*— Autonomous linear generator, Discrete - time PID observer, Francis equation, The internal model principle.**

I. Introduction

The challenge of compensating for external disturbances is a crucial and practical aspect of automatic control theory [1-3]. A commonly adopted approach relies on the internal model principle (IMP) [1-5], in which external signals are represented as the output of an autonomous linear generator. The IMP proves to be an efficient approach for handling external exosignals that need to be rejected or tracked. This principle involves modeling the reference signal to be tracked or the disturbance to be rejected as the output of an autonomous dynamic system, referred to as an exosystem, driven by the nonzero initial conditions of its state. By appropriately replicating the exosystem model within the closed-loop system's structure, it becomes possible to achieve asymptotic tracking of the reference signal or completely neutralize the effect of the disturbance.

If the parameters of the control system and the exosystem are known, the output regulation problem can be addressed using a nonadaptive controller with fixed precalculated parameters [6], [7]. In this paper, we focus on the nonadaptive implementation of the external disturbance compensation problem for tracking control. Regarding this issue, study [2] provided a comprehensive presentation of the implementation of the control law and the construction of a state observer based on the internal model principle. However, the investigated system was a continuous-time single-input single-output linear system, considering the influence of input disturbances. Expanding on this research, study [1] proposed a control law for continuous-time linear MIMO systems subject to both input and output disturbances, under the assumption that both the system states and external disturbance states are directly measurable. As a result, a state observer was not constructed in that study. To estimate the system states for external disturbance compensation, observers with unknown input signals [8-10] have been studied and developed. However, these studies have only been applied to continuous-time MIMO systems affected by input disturbances. Currently, disturbance compensation methods have been extended to consider discrete MIMO systems [11-12]. However, these methods also assume that the system states can be directly measured. Therefore, the application of this approach to a class of discrete-time multichannel systems affected by both input and output disturbances remains an open question.

In this study, we introduce a novel approach for designing a control law to compensate for external disturbances in the tracking control process of discrete-time MIMO systems, which are affected by both input and output disturbances. The challenge arises from the fact that the system states and external disturbances are not directly observable. To overcome this limitation, a discrete-time PID observer is constructed to estimate the components of the state vector. Utilizing these estimates, a control strategy is formulated based on Francis' equation to ensure effective tracking and disturbance compensation.

The structure of the paper is as follows: Section 2 presents the problem formulation, while Section 3 introduces the design of the full-order state observer. The controller design for disturbance compensation is presented in Section 4. Section 5 presents the computation and simulation results using MATLAB Simulink. Finally, the conclusions of the study are presented in Section 6.

II. Problem statement

Consider the discrete-time linear MIMO system affected by input disturbance and output disturbance as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + D\upsilon(k) \\ y(k) = Cx(k) + R\upsilon(k) \end{cases} \tag{1}$$

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where $x(k) \in \mathbb{R}^n$ is the unmeasured state vector of plant; $u(k) \in \mathbb{R}^m$ is the vector of control signal; $y(k) \in \mathbb{R}^m$ is the vector of regulated variables; $\upsilon(k) \in \mathbb{R}^p$ is the external disturbance; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{n\times p}$ and $R \in \mathbb{R}^{m\times p}$ are known constant matrices, $n \geq m$.

The objective is to design a control law that satisfies the following requirements

$$\lim_{k\to\infty} \left| y(k) - g(k) \right| \leq O(T) \tag{2}$$

where $g(k)$ is the reference signal, T is the sampling time; $O(T)$ means that $\lim_{k\to\infty} \frac{O(T)}{T} = const$, this means that the tracking error depends on the sampling time.

This paper is based on the following assumptions:

Assumption 1. The known constant matrix pairs $(A,\ B)$ and $(A,\ C)$ are respectively controllable and observable.

Assumption 2. The reference signal g and the external disturbance υ are both bounded and can be expressed as the output of a linear autonomous generator [1].

$$\begin{cases} \xi(k+1) = \Gamma\xi(k) \\ g(k) = Q\xi(k) \\ \upsilon(k) = H\xi(k) \end{cases} \tag{3}$$

where $\xi(k) \in \mathbb{R}^q$ is the state vector of the external signal model, $\Gamma \in \mathbb{R}^{q\times q}$, $Q \in \mathbb{R}^{m\times q}$, $H \in \mathbb{R}^{p\times q}$ are known constant matrices.

Assumption 3. These matrices A, C satisfy [13]

$$Rank\left(\begin{bmatrix} A - I_n & G \\ C - RR^{+}C & 0 \end{bmatrix}\right) = n + k.$$

Here, $R^{+} = (R^T R)^{-1} R^T \in \mathbb{R}^{l\times m}$ is the pseudo-inverse matrix of R such that $R^{+}R = I_l$; $G = (\frac{1}{T} ln(A))^{-1}(A - I_n)D$ with $rank(G) = k$ and $rank(R) = l$.

Assumption 4. The system, in terms of the relationship between the output and disturbance, exhibits a minimum - phase behavior. Consequently, it follows [13] that.

$$Rank\left(\begin{bmatrix} zI_n - A & 0 \\ C & R \end{bmatrix}\right) = n + l \quad \forall z \in \mathrm{C},\ |z| \geq 1.$$

Assumption 5. The sampling time T is sufficiently small, ensuring that the disturbance remains nearly constant between consecutive sampling points.

Remark 1. Assumption 1 and Assumption 2 are the conditions for synthesizing the control law based on the Francis equation to ensure disturbance compensation and that the output signal follows the reference signal. Assumption 3, Assumption 4, and Assumption 5 are used to construct the PID observer for estimating the states of the object and external disturbances, with an estimation error smaller than a size of $O(T)$ [13].

Fig. 1 illustrates a schematic of the closed-loop system for the proposed approach in the case of a stable system.

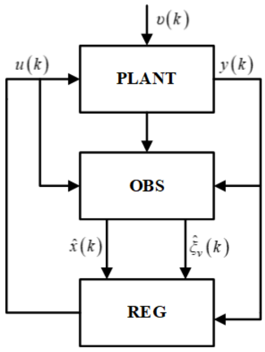

Fig. 1 - Design of the closed-loop system (OBS represents the general observation unit: state observer $\hat{x}(k)$ and the state observer of external disturbances $\xi_v(k)$, REG is regulator).

III. Design of observer

As the states of the plant and the external disturbances are not directly measurable, a full-order state observer must be designed. Given the assumption that the system is simultaneously affected by both input and output disturbances, a discrete-time PID state observer [13] is therefore chosen for construction.

To construct a discrete-time PID state observer for the system (1), let $f(k) = D\upsilon(k)$, we can rewrite system (1) in the form of a descriptor system

$$\begin{cases} \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k+1) \\ \upsilon(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -R^{+}C & -I_l \end{bmatrix} \begin{bmatrix} x(k) \\ \upsilon(k) \end{bmatrix} \\ \qquad + \begin{bmatrix} B & 0 \\ 0 & R^{+} \end{bmatrix} \begin{bmatrix} u(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} f(k) \\ 0 \end{bmatrix}. \\ y(k) = \begin{bmatrix} C & R \end{bmatrix} \begin{bmatrix} x(k) \\ \upsilon(k) \end{bmatrix} \end{cases} \tag{4}$$

Define the augmented vector z as $z = \begin{bmatrix} x \\ \upsilon \end{bmatrix} \in \mathbb{R}^{n+l}$ and rewrite system (4) as follows

$$\begin{cases} E_{\Sigma} z(k+1) = A_{\Sigma} z(k) + B_{\Sigma}\Phi(k) + f_{\Sigma}(k) \\ y(k) = C_{\Sigma} z(k) \end{cases}, \tag{5}$$

where the system matrices are defined as

$$E_{\Sigma} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+l)\times(n+l)}\ B_{\Sigma} = \begin{bmatrix} B & 0 \\ 0 & R^{+} \end{bmatrix} \in \mathbb{R}^{(n+l)\times(m+m)}$$

$$A_{\Sigma} = \begin{bmatrix} A & 0 \\ -R^{+}C & -I_l \end{bmatrix} \in \mathbb{R}^{(n+l)\times(n+l)},\ \Phi = \begin{bmatrix} u \\ y \end{bmatrix} \in \mathbb{R}^{m+m},$$

$$f_{\Sigma} = \begin{bmatrix} f \\ 0 \end{bmatrix} \in \mathbb{R}^{n+l},\ C_{\Sigma} = \begin{bmatrix} C & R \end{bmatrix} \in \mathbb{R}^{m\text{x}(n+l)}.$$

Let $K_{\Sigma} = \begin{bmatrix} 0 \\ R^{+} \end{bmatrix} \in \mathbb{R}^{(n+l)\text{x}m}$, $G_{\Sigma} = \begin{bmatrix} G \\ 0 \end{bmatrix} \in \mathbb{R}^{(n+l)\text{x}k}$, $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{\upsilon} \end{bmatrix} \in \mathbb{R}^{n+l}$. And construct the PID observer for the descriptor system in equation (5) as follows

$$\begin{cases} \left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)\eta\left(k+1\right) = \left(A_{\Sigma} - NC_{\Sigma}\right)\eta\left(k\right) \\ \quad + \left(\left(A_{\Sigma} - NC_{\Sigma}\right)\left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)^{-1} K_{\Sigma} + N\right) y\left(k\right) \\ \quad + B_{\Sigma}\Phi\left(k\right) + G_{\Sigma}q\left(k\right) \\ q\left(k+1\right) = q\left(k\right) + \bar{L}\left(y\left(k\right) - C_{\Sigma}\hat{z}(k)\right) \\ \hat{z}(k) = \eta\left(k\right) + \left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)^{-1} K_{\Sigma}y\left(k\right) \end{cases}, \tag{6}$$

where $\hat{z} \in \mathbb{R}^{n+l}$, $q \in \mathbb{R}^{k}$, $N \in \mathbb{R}^{n\text{x}m}$ and $\bar{L} \in \mathbb{R}^{kxm}$.

Proposition 1. The matrices N and $\bar{L}$ can be computed as follows

$$N = \left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)L_1; \quad \bar{L} = L_2.$$

The matrices L_1 and L_2 are obtained from the equation

$$L = A_w P_w C_w^T \left(R_w + C_w P_w C_w^T\right)^{-1},$$

where $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \in \mathbb{R}^{(n+l+k)\text{x}m}$ and $P_w \in \mathbb{R}^{(n+l+k)\text{x}(n+l+k)} > 0$ is obtained from the following discrete algebraic Riccati equation

$$Q_w = P_w - A_w^T P_w A_w + A_w P_w C_w^T \left(A_w P_w C_w^T + R_w\right)^{-1} C_w P_w A_w^T.$$

Here, $Q_w \in \mathbb{R}^{(n+l+k)\text{x}(n+l+k)} > 0$ and $R_w \in \mathbb{R}^{mxm} > 0$,

$$A_w = \begin{bmatrix} \left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)^{-1} A_{\Sigma} & \left(E_{\Sigma} + K_{\Sigma}C_{\Sigma}\right)^{-1} G_{\Sigma} \\ 0 & I_k \end{bmatrix},$$

$A_w \in \mathbb{R}^{(n+l+k)\text{x}(n+l+k)}$, $C_w = \begin{bmatrix} C_{\Sigma} & 0 \end{bmatrix} \in \mathbb{R}^{m\text{x}(n+l+k)}$.

Proof: See [13].

At this stage, we have obtained estimates for both the system state and the external disturbance. The next step is to refine the external disturbance state using the previously derived disturbance estimation.

Suppose the reference signal and the external disturbance signal are represented by the following models:

$$\begin{cases} \xi_g\left(k+1\right) = \Gamma_g \xi_g\left(k\right) \\ g\left(k\right) = Q_g \xi_g\left(k\right) \end{cases}, \tag{7}$$

$$\begin{cases} \xi_v\left(k+1\right) = \Gamma_v \xi_v\left(k\right) \\ \upsilon\left(k\right) = H_v \xi_v\left(k\right) \end{cases}, \tag{8}$$

where $\xi_g\left(k\right) \in \mathbb{R}^{q_v}$ is the state vector of the reference model, $\xi_v\left(k\right) \in \mathbb{R}^{q_v}$ is the state vector of the disturbance model; $\Gamma_g \in \mathbb{R}^{q_g \times q_g}$, $Q_g \in \mathbb{R}^{m \times q_g}$, $\Gamma_v \in \mathbb{R}^{q_v \times q_v}$, $H_v \in \mathbb{R}^{p \times q_v}$ are known constant matrices.

Proposition 2. The state estimation of the external disturbance is obtained as:

$$\begin{cases} \hat{\xi}_v\left(k\right) = M_d^{-1}\bar{\xi}_v\left(k\right) \\ \bar{\xi}_v\left(k+1\right) = G_d\bar{\xi}_v\left(k\right) + L_d\upsilon(k) \end{cases}. \tag{9}$$

Here, $M_d \in \mathbb{R}^{q_v \times q_v}$ is a solution of the Sylvester equation

$$M_d\Gamma_v - G_d M_d = L_d H_v \tag{10}$$

$G_d \in \mathbb{R}^{q_v \times q_v}$ is an arbitrary Hurwitz matrix and a constant vector $L_d \in \mathbb{R}^{q_v}$ such that the pair (G_d, L_d) is controllable.

Proof: See [1].

IV. DESIGN OF COMPENSATOR

To achieve disturbance compensation in the tracking control process, we design a regulator following the approach in [1]. The output error is defined as follows:

$$\varepsilon\left(k\right) = y\left(k\right) - g\left(k\right). \tag{11}$$

Using equations (1) and (3), we derive:

$$x\left(k+1\right) = Ax\left(k\right) + Bu\left(k\right) + DH\xi\left(k\right), \tag{12}$$

$$\varepsilon\left(k\right) = Cx\left(k\right) + RH\xi\left(k\right) - Q\xi\left(k\right). \tag{13}$$

Next, we transform the coordinates of the exosignals into the plant state vector's coordinate frame using the transformation matrix M. The tracking error of the plant state is then expressed as:

$$e\left(k\right) = x\left(k\right) - M\xi\left(k\right). \tag{14}$$

Given that $x\left(k\right) = e\left(k\right) + M\xi\left(k\right)$, (13) takes the form

$$\varepsilon\left(k\right) = Ce\left(k\right) + \left(CM + RH - Q\right)\xi\left(k\right). \tag{15}$$

Considering the difference equation of e, in view of (3) and (12), we have

$$\begin{aligned} e\left(k+1\right) &= Ax\left(k\right) + Bu\left(k\right) + DH\xi\left(k\right) - M\Gamma\xi\left(k\right) \\ &= Ae\left(k\right) + Bu\left(k\right) + (AM - M\Gamma + DH)\xi\left(k\right). \end{aligned} \tag{16}$$

Let us assume that there exist matrices $M \in \mathbb{R}^{n \times q}$, and $\Theta_e \in \mathbb{R}^{m \times q}$, which solve the following matrix equation, refered to as Francis or, also, the regulator equation:

$$AM - M\Gamma + DH = B\Theta_e, \tag{17}$$

$$CM = Q - RH. \tag{18}$$

Then the plant model expressed in the new variables e and ε takes the form:

$$e\left(k+1\right) = Ae\left(k\right) + B\left(u\left(k\right) + \Theta_e\xi\left(k\right)\right), \tag{19}$$

$$\varepsilon\left(k\right) = Ce\left(k\right). \tag{20}$$

Theorem 1. If the matrix pair $(A,\ B)$ is stabilizable and there exists a solution pair $(M,\ \Theta_e)$ for the matrix equation (17, 18). The following control law [1] ensures that the tracking error $\varepsilon(k)$ satisfies the control objective (2)

$$u(k) = -\Theta_e\hat{\xi}(k) + u_s(k). \tag{21}$$

Here, $u_s(k) = -K\left(\hat{x}(k) - M\hat{\xi}(k)\right)$ with K chosen such that $A - BK$ is Hurwitz.

Proof:

Substituting (21) into (19), we obtain the closed-loop error model

$$\begin{aligned} e(k+1) = {} & Ae(k) - B\Theta_e\left(\xi(k) - \hat{\xi}(k)\right) \\ & - BK\left(\hat{x}(k) - M\hat{\xi}(k)\right) \end{aligned} \tag{22}$$

Let $\tilde{x}(k) = x(k) - \hat{x}(k)$ and $\tilde{\xi}(k) = \xi(k) - \hat{\xi}(k)$ be the estimation errors of the observer. Equation (22) can be reformulated as follows:

$$e(k+1) = (A - BK)e(k) - B\Theta_e\tilde{\xi}(k) - BK\tilde{x}(k). \tag{23}$$

Since $A - BK$ is Hurwitz and the state estimation of the object and the external disturbance has an error with a value smaller than the size of $O(T)$ [13], we obtain from equation (20) that:

$$\lim_{k\to\infty}\left|\varepsilon(k)\right| \le O(T). \tag{24}$$

Remark 2. Equations (17) and (18) can be separated into two independent equations for the reference signal and the disturbance signal, respectively, as follows [1]

$$\begin{aligned} \begin{bmatrix} B\Theta_g & B\Theta_v \end{bmatrix} = {} & A\begin{bmatrix} M_g & M_v \end{bmatrix} \\ & -\begin{bmatrix} M_g & M_v \end{bmatrix}\begin{bmatrix} \Gamma_g & 0 \\ 0 & \Gamma_v \end{bmatrix} + D\begin{bmatrix} 0 & H_v \end{bmatrix}, \end{aligned} \tag{25}$$

$$C\begin{bmatrix} M_g & M_v \end{bmatrix} = \begin{bmatrix} Q_g & 0 \end{bmatrix} - \begin{bmatrix} 0 & RH_v \end{bmatrix}. \tag{26}$$

where $M_g \in \mathbb{R}^{n\times q_g}$, $M_v \in \mathbb{R}^{n\times q_v}$, $\Theta_g \in \mathbb{R}^{m\times q_g}$ and $\Theta_v \in \mathbb{R}^{m\times q_v}$.

V. Numerical example

Consider the continuous-time system, represented as follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ -4 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \upsilon(t) \\ y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \upsilon(t) \end{cases}.$$

Both the reference g and the disturbance υ are referred to as exosignals

$$g(t) = \begin{vmatrix} g_1(t) \\ g_2(t) \end{vmatrix} = \begin{vmatrix} 2\sin(\omega_1 t) \\ 4 \end{vmatrix};\ \upsilon(t) = 1.5\sin(\omega_2 t).$$

Transform to discrete system with T = 0.005 s, we obtain

$$\begin{cases} x(k+1) = \begin{bmatrix} 1 & 0.005 & 0 \\ -0.0199 & 0.9851 & 0 \\ 0 & 0 & 0.995 \end{bmatrix} x(k) \\ \qquad + \begin{bmatrix} 0 & 0 \\ 0.005 & 0 \\ 0 & 0.05 \end{bmatrix} u(k) + \begin{bmatrix} 0.005 \\ 0 \\ 0.005 \end{bmatrix} \upsilon(k). \\ y(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \upsilon(k) \end{cases}$$

The matrices of the exosignals with $\omega_1 = 1$, $\omega_2 = 0.5$ in the discrete system are as follows:

$$\Gamma_g = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \Gamma_v = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix},$$

$$Q_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ H_v = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

By following the design procedure outlined in Section 3 and selecting $Q_w = I_5$ and $R_w = I_2$, we derive the gain matrices L_1, $\bar{L}$ and N as follows:

$$L_1 = \begin{bmatrix} 0.0485 & -0.0017 & -0.164 & -0.7716 \\ 0.7556 & -0.0096 & 0.758 & 0.7834 \end{bmatrix}^T,$$

$$\bar{L} = \begin{bmatrix} -0.0719 & 0.3043 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.0485 & -0.0017 & -0.164 & -0.7231 \\ 0.7556 & -0.0096 & 0.758 & -0.0278 \end{bmatrix}^T.$$

By following the design procedure in Section 4, we can derive the gain matrices M_g, M_v, Θ_g and Θ_v as follows:

$$M_g = \begin{bmatrix} 1 & 0 & 0 \\ -199.9971 & 199.9996 & 0 \\ -1 & 0 & 1 \end{bmatrix},$$

$$M_v = \begin{bmatrix} 0 & -1 \\ 198.4974 & -199.4968 \\ 0 & 1 \end{bmatrix},$$

$$\Theta_g = \begin{bmatrix} 598.5012 & -601.5013 & 0 \\ -199.5004 & 200.5004 & -1 \end{bmatrix},$$

$$\Theta_v = \begin{bmatrix} -798.8733 & 805.6202 \\ 201.5004 & -201.4992 \end{bmatrix}.$$

The simulation results are presented from Fig.2 to Fig.4.

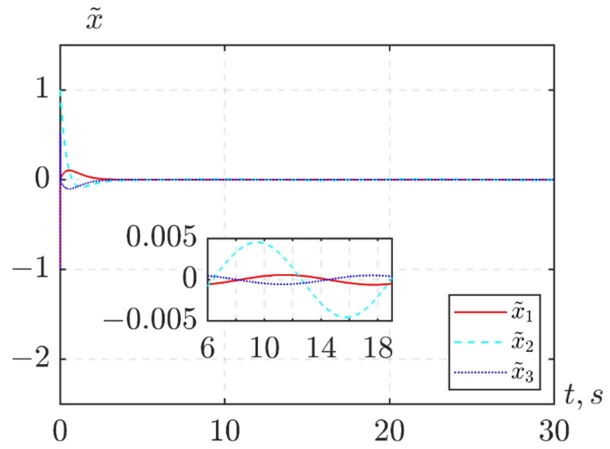

Fig. 2 - State estimation error of plant

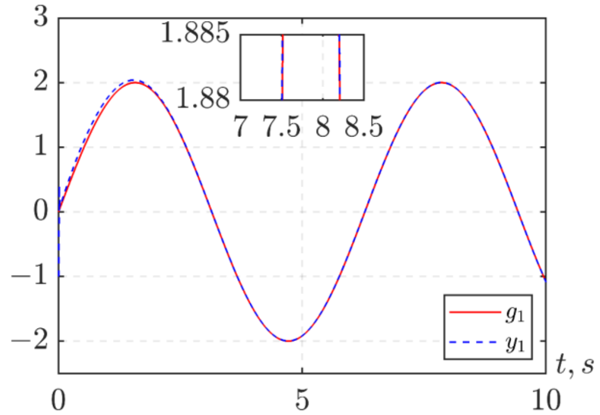

Fig. 3 - The graphs of input g_1 and output signal y_1

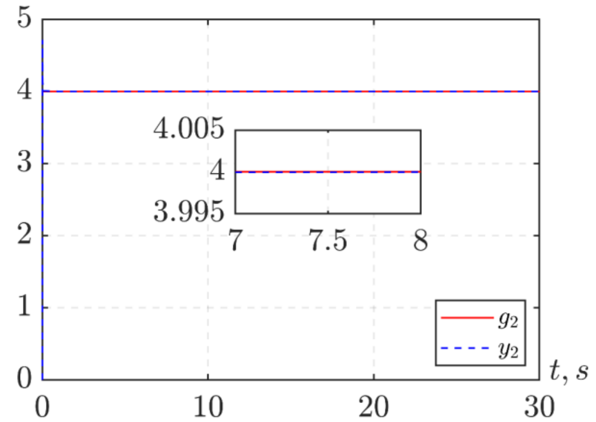

Fig. 4 - The graphs of input g_2 and output signal y_2

The simulation results in Fig. 2 show that the state estimation of the object has an error with a value smaller than a size of $O(T)$. However, according to Fig. 3 and 4, it can be observed that the system's output signal effectively tracks the reference signal. The evaluation of the simulation results shows that the control law guarantees precise tracking and effective disturbance compensation when the sampling time is adequately small.

VI. Conclusion

This study proposes a method for designing controllers that ensure tracking and disturbance compensation in discrete-time linear MIMO systems subject to both input and output disturbances, while accounting for the fact that neither the system states nor the disturbance states are directly observable. The simulation results indicate that although the PID observer does not achieve exact asymptotic estimation, it can ensure that the estimation error remains within $O(T)$ when the disturbance between two consecutive sampling points does not vary significantly. Additionally, the control law effectively ensures tracking and compensates for external disturbances. In future work, the authors will investigate discrete-time MIMO systems in tracking mode, considering external signals with unknown parameters.

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